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DOES NNP GROWTH INDICATE WELFARE IMPROVEMENT?

GEIR B. ASHEIM AND MARTIN L. WEITZMAN

Abstract. We show that instantaneous increases in real NNP over time are an accurate indicator of true dynamic welfare improvements. The framework of the paper is the standard multisector optimal growth model. The result highlights a connection between the theory of green (or comprehensive) national income accounting and the theory of real price indices.

1. Introduction

It has been known for some time now that the current-value Hamiltonian of an optimal growth problem represents in welfare terms the level of stationary-equivalent future utility. It is also apparent that a current-value Hamiltonian is essentially comprehensive NNP expressed in utility units. Somewhat less apparent is how actually to use the above insights in a world where measurable NNP is expressed in monetary (rather than utility) units.

In this paper we show that welfare is increasing instantaneously over time if and only if real NNP is increasing instantaneously over time. Thus, contrary to some opinions that have been expressed in the literature, time changes in real NNP mirror accurately changes in dynamic welfare, at least locally.

The framework used for establishing the above result is the standard multisector optimal growth model with time-invariant technology. The result may be useful because it shows an intriguing connection between the theory of green (or comprehensive) accounting and the theory of price deflators. In particular, the paper establishes a new conceptual link between the Divisia index of real consumption prices and dynamic welfare evaluation.

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2. The model

Let the vector $\mathbf{C}$ represent a m-dimensional fully-disaggregated consumption bundle, containing everything that influences current *well being*, including environmental amenities and other externalities. (Supplied labor corresponds to negative components.) Current consumption is presumed to be fully observable, along with its associated m-vector of efficiency prices. For any consumption-flow $\{\mathbf{C}(t)\}$, overall intertemporal *welfare* is measured by

$$W(\{\mathbf{C}(t)\}) := \int_0^\infty e^{-\rho t} U(\mathbf{C}(t)) dt \,, \tag{1}$$

where U is a given concave and non-decreasing utility function with continuous second derivatives, while ρ is a given utility discount rate.

There are n capital goods, including stocks of natural resources, environmental assets, human capital (like education and knowledge capital accumulated from R&D-like activities), and other nonorthodox forms. The stock of capital of type j $(1 \leq j \leq n)$ at time t is denoted $K_j(t)$, and its corresponding net investment flow is $I_j(t) = \dot{K}_j(t)$. The n-vector $\mathbf{K} = \{K_j\}$ denotes all capital stocks, while $\mathbf{I} = \{I_j\}$ stands for the corresponding n-vector of net investments. The net investment flow of a natural capital asset is negative if the overall extraction rate exceeds the replacement rate.

We are imagining an idealized world where the coverage of capital goods is so comprehensive, and the national accounting system so complete, that there remain no unaccounted-for residual growth factors. Thus, *all* sources of future growth are fully "accounted-for" as investments that are valued at their efficiency prices and included in national product. Formally, the $(m + n)$–dimensional attainable-possibilities set at any time t is a function S only of the capital stocks $\mathbf{K}(t)$ at that time. Therefore, the consumption-investment pair $(\mathbf{C}(t), \mathbf{I}(t))$ is attainable at time t if and only if

$$(\mathbf{C}(t), \mathbf{I}(t)) \in S(\mathbf{K}(t)) \,. \tag{2}$$

The set of attainable possibilities $S(\mathbf{K})$ is presumed to be convex.

3. Optimal growth

Consider the standard optimal growth problem: Maximize (1) subject to constraints (2) and $\dot{\mathbf{K}}(t) = \mathbf{I}(t)$, and obeying the initial condition $\mathbf{K}(0) = \mathbf{K}_0$, where $\mathbf{K}_0$ is given. In what follows, it is assumed for simplicity that an optimal solution exists and is unique. Let $\{\mathbf{C}^*(t)\}$, $\{\mathbf{I}^*(t)\}$, and $\{\mathbf{K}^*(t)\}$

represent the optimal trajectories of consumption, investment and capital. Since the attainable-possibilities set at any time is a function only of the capital stocks at that time, it follows that maximized welfare at time t, $W^*(t)$, is a function only of the capital stocks at time t:

$$W^*(t) = \mathcal{W}(\mathbf{K}^*(t)) := \int_t^\infty e^{-\rho(s-t)} U(\mathbf{C}^*(s)) ds \,.$$

Let $\{\mathbf{\Psi}(t)\}$ represent the trajectory of the dual vector of shadow investment prices, relative to utility being the numeraire. Applying the maximum principle of control theory to the above optimization problem, and letting the current-value Hamiltonian be given by

$$H(\mathbf{C}, \mathbf{I}; \mathbf{\Psi}) = U(\mathbf{C}) + \mathbf{\Psi}\mathbf{I} \,,$$

we obtain that, at each t, $(\mathbf{C}^*(t), \mathbf{I}^*(t))$ maximizes $H(\mathbf{C}, \mathbf{I}; \mathbf{\Psi}(t))$ subject to $(\mathbf{C}(t), \mathbf{I}(t)) \in S(\mathbf{K}^*(t))$:

$$\begin{aligned} H^*(t) = \mathcal{H}(\mathbf{K}^*(t), \mathbf{\Psi}(t)) &:= \max_{(\mathbf{C},\mathbf{I}) \in S(\mathbf{K}^*(t))} H(\mathbf{C}, \mathbf{I}; \mathbf{\Psi}(t)) \\ &= U(\mathbf{C}^*(t)) + \mathbf{\Psi}(t)\mathbf{I}^*(t) \,. \end{aligned} \tag{3}$$

Refer to $\mathbf{\Psi}(t)\mathbf{I}^*(t)$ as the *value of net investments*. Furthermore, we have as co-state differential equations that

$$\nabla \mathcal{H}_{\mathbf{K}}(\mathbf{K}^*(t), \mathbf{\Psi}(t)) = \rho \mathbf{\Psi}(t) - \dot{\mathbf{\Psi}}(t) \,, \tag{4}$$

where ∇ denotes a vector of partial derivatives. Finally, the relevant transversality conditions are $e^{-\rho T}\mathbf{\Psi}(T)\mathbf{K}^*(T) \to 0$ and $e^{-\rho T}\mathcal{H}(\mathbf{K}^*(T), \mathbf{\Psi}(T)) \to 0$ as $T \to \infty$, implying that $e^{-\rho T}\mathbf{\Psi}(T)\mathbf{I}^*(T) \to 0$ as $T \to \infty$ (cf. Michel [4]).

There is a basic result – first established by Dixit et al. [2] – that is of fundamental importance for the analysis that follows.

Lemma 1. *Under the given assumptions,*

$$\nabla U(\mathbf{C}^*(t))\dot{\mathbf{C}}^*(t) + d(\mathbf{\Psi}(t)\mathbf{I}^*(t))/dt = \rho \mathbf{\Psi}(t)\mathbf{I}^*(t)$$

holds at any t.

Proof. By (3), (4), and the envelope theorem, it follows that

$$\dot{H}^* = \nabla \mathcal{H}_{\mathbf{K}} \mathbf{I}^* + \nabla \mathcal{H}_{\mathbf{\Psi}} \dot{\mathbf{\Psi}} = (\rho \mathbf{\Psi} - \dot{\mathbf{\Psi}})\mathbf{I}^* + \dot{\mathbf{\Psi}}\mathbf{I}^* = \rho \mathbf{\Psi}\mathbf{I}^* \tag{5}$$

However, (3) also directly implies that

$$\dot{H}^* = \nabla U(\mathbf{C}^*)\dot{\mathbf{C}}^* + d(\mathbf{\Psi}\mathbf{I}^*)/dt \tag{6}$$

The lemma is obtained by combining (5) and (6). □

Lemma 1 implies the following result, noted by e.g. Dasgupta & Mäler [1] and Pemberton & Ulph [5].

Proposition 1. *Under the given assumptions,*

$$\dot{W}^*(t) = \mathbf{\Psi}(t)\mathbf{I}^*(t)$$

holds at any t.

Proof. The proposition is obtained through integration since

$$\begin{aligned}\dot{W}^*(t) &= -U(\mathbf{C}^*(t)) + \rho\int_t^\infty e^{-\rho(s-t)}U(\mathbf{C}^*)ds \\ &= \int_t^\infty e^{-\rho(s-t)}\nabla U(\mathbf{C}^*)\dot{\mathbf{C}}^* ds = -\int_t^\infty \Big(d(e^{-\rho(s-t)}\mathbf{\Psi}\mathbf{I}^*)/ds\Big)\, ds\,,\end{aligned}$$

where the second equality follows by integrating by parts, and the third equality follows from Lemma 1. □

This result means that the value of net investments has the following welfare significance: Maximized welfare is increasing if and only if $\mathbf{\Psi}\mathbf{I}^*$ is positive.

Measurable comprehensive net national product (NNP) is frequently identified in the literature with the "linearized" Hamiltonian (cf. e.g. Hartwick [3]), being the sum of the "value of consumption" and the value of net investments, measured in monetary units. While Prop. 1 implies that welfare is increasing if and only if measurable NNP exceeds the value of consumption, this is a different kind of welfare significance than the one sought by Weitzman [7], where higher welfare is indicated by higher NNP. The latter interpretation would translate here into a result that welfare is increasing along the time axis if and only if measurable NNP is also increasing. Can such a result be established?

4. NNP in nominal prices

If the optimal growth trajectory is realized through an intertemporal competitive equilibrium, market prices will be expressed in monetary units. Neither the vector of marginal utilities, $\nabla U(\mathbf{C}^*)$, nor the vector of investment prices in utility units, $\mathbf{\Psi}$, are directly observable. Rather, what may be observed directly are *nominal* prices at time t for consumption goods and investment flows, given respectively by

$$\begin{aligned}\mathbf{p}(t) &= \nabla U(\mathbf{C}^*(t))/\lambda(t) \\ \mathbf{q}(t) &= \mathbf{\Psi}(t)/\lambda(t)\,,\end{aligned}$$

and a *nominal* interest rate at time t, $r(t)$, given by

$$r(t) = \rho - \tfrac{\dot{\lambda}(t)}{\lambda(t)}\,,$$

where $\lambda(t) > 0$ is the not-directly-observable marginal utility of current expenditures, which may depend on the "quantity of money" at time t.

At any time t, consumers maximize utility and producers maximize profit:

$$\mathbf{C}^*(t) \text{ maximizes } U(\mathbf{C}) - \lambda(t)\mathbf{p}(t)\mathbf{C}\,, \tag{7}$$

$$\begin{aligned} (\mathbf{C}^*(t), \mathbf{I}^*(t), \mathbf{K}^*(t)) \text{ maximizes } & \mathbf{p}(t)\mathbf{C} + \mathbf{q}(t)\mathbf{I} - (r(t)\mathbf{q}(t) - \dot{\mathbf{q}}(t))\mathbf{K} \\ & \text{over all } (\mathbf{C}, \mathbf{I}, \mathbf{K}) \text{ satisfying } (\mathbf{C}, \mathbf{I}) \in S(\mathbf{K})\,, \end{aligned} \tag{8}$$

where $r(t)q_j(t) - \dot{q}_j(t)$ is the cost of holding one unit of capital good j. We have that (7) follows from the concavity of U, while (8) follows from the convexity of $S(\mathbf{K})$ for any $\mathbf{K}$, the maximum principle, and the property that $\nabla\mathcal{H}_{\mathbf{K}} = \rho\mathbf{\Psi} - \dot{\mathbf{\Psi}} = \rho\lambda\mathbf{q} - \dot{\lambda}\mathbf{q} - \lambda\dot{\mathbf{q}} = \lambda(r\mathbf{q} - \dot{\mathbf{q}})$. This latter property also means that Lemma 1, expressed in nominal prices, yields

$$\mathbf{p}(t)\dot{\mathbf{C}}^*(t) + d(\mathbf{q}(t)\mathbf{I}^*(t))/dt = r(t)\mathbf{q}(t)\mathbf{I}^*(t)\,. \tag{9}$$

Define comprehensive *NNP in nominal prices*, $y(t)$, as the sum of the nominal value of consumption and the nominal value of net investments:

$$y(t) := \mathbf{p}(t)\mathbf{C}^*(t) + \mathbf{q}(t)\mathbf{I}^*(t)\,.$$

It follows from Prop. 1 that maximized welfare is increasing if and only if NNP exceeds the value of consumption:

$$\dot{W}^*(t) > 0 \;\Leftrightarrow\; y(t) - \mathbf{p}(t)\mathbf{C}^*(t) = \mathbf{q}(t)\mathbf{I}^*(t) > 0\,.$$

However, since the level of NNP in nominal prices at t depends on $\lambda(t)$, and $\lambda(t)$ is arbitrary, the condition that $\dot{y}(t) > 0$ cannot signify welfare improvement. For a *change* in NNP (as opposed to a comparison of NNP with the value of consumption) to indicate a change in welfare, NNP must be measured in *real* prices. How then should NNP in real prices be determined?

5. NNP in real prices and local comparisons

In this section we build upon a finding by Sefton & Weale [6] that a Divisia consumption price index is of essential importance when expressing comprehensive NNP in real prices. By using such a price index, we show that a claim made by Dasgupta & Mäler ([1], Sect. 7.1) – namely that comprehensive real NNP cannot be used for intertemporal welfare comparisons – is incorrect.

The application of a price index $\{\pi(t)\}$ turns nominal prices $\{\mathbf{p}(t), \mathbf{q}(t)\}$ into *real* prices $\{\mathbf{P}(t), \mathbf{Q}(t)\}$,

$$\mathbf{P}(t) = \mathbf{p}(t)/\pi(t)$$
$$\mathbf{Q}(t) = \mathbf{q}(t)/\pi(t)\,,$$

implying that the *real* interest rate, $R(t)$, at time t is given by

$$R(t) = r(t) - \tfrac{\dot{\pi}(t)}{\pi(t)}\,.$$

A *Divisia* price index satisfies $\dot{\mathbf{P}}\mathbf{C}^* = 0$ at each t, implying that

$$0 = \dot{\mathbf{P}}\mathbf{C}^* = \frac{d}{dt}\left(\frac{\mathbf{p}}{\pi}\right)\mathbf{C}^* = \frac{\pi\dot{\mathbf{p}}\mathbf{C}^* - \dot{\pi}\mathbf{p}\mathbf{C}^*}{\pi^2}\,,$$

or, equivalently,

$$\frac{\dot{\pi}(t)}{\pi(t)} = \frac{\dot{\mathbf{p}}(t)\mathbf{C}^*(t)}{\mathbf{p}(t)\mathbf{C}^*(t)}\,.$$

Define comprehensive *NNP in real Divisia prices*, $Y(t)$, as the sum of the real value of consumption and the real value of net investments:

$$Y(t) := \mathbf{P}(t)\mathbf{C}^*(t) + \mathbf{Q}(t)\mathbf{I}^*(t)\,.$$

Proposition 2. *Under the given assumptions,*

$$\dot{Y}(t) = R(t)\left(Y(t) - \mathbf{P}(t)\mathbf{C}^*(t)\right)$$

holds at any t.

Proof. It follows from the definition of $Y(t)$ that

$$\begin{aligned}\dot{Y} = d\left(\mathbf{P}\mathbf{C}^* + \mathbf{Q}\mathbf{I}^*\right)/dt &= \mathbf{P}\dot{\mathbf{C}}^* + d(\mathbf{Q}\mathbf{I}^*)/dt \\ &= R\mathbf{Q}\mathbf{I}^* = R\left(Y - \mathbf{P}\mathbf{C}^*\right)\,,\end{aligned}$$

where the second equality follows since $\dot{\mathbf{P}}\mathbf{C}^* = 0$, and the third equation is obtained since (9) holds also for $\{\mathbf{P}(t), \mathbf{Q}(t)\}$ and $\{R(t)\}$. □

Combining Props. 1 and 2, we have the main result of this paper.

Proposition 3. *Provided that the real interest rate is positive, growth in real NNP means that welfare is increasing.*

6. Discussion

The case of a so-called "cake-eating" economy – where no production takes place and consumption at time t equals the extraction at time t of a non-renewable and finite natural resource – might seem to imply that a theorem like Prop. 2 cannot be established. A cake-eating economy's comprehensive NNP is identical to zero, since consumption at each point in time equals extraction, and thus, the value of consumption and the value of net investment add up to zero. How can this result be reconciled with Prop. 2? The key issue here is that the real interest rate, R, in a "cake-eating" economy is identical to zero. Hence, even though a negative value of net investment in the resource (by Prop. 1) means that welfare is decreasing, by Prop. 2 comprehensive NNP is constant and equal to zero. Note that the paradox vanishes for any economy where $R > 0$.

The real prices used in Prop. 2 are derived through a Divisia *consumption* price index. Hence, although welfare improvement is indicated by real growth in NNP – comprising the value of both consumption and net investments – only the consumption goods (including supplied labor as negative components) should be used as quantity weights in the price index. In fact, since $\nabla U(\mathbf{C}^*) = \lambda\pi\mathbf{P}$ and $\dot{\mathbf{P}}\mathbf{C}^* = 0$, real Divisia prices satisfy the condition that increased instantaneous well being is indicated by growth in real consumption expenditures:

$$d\left(U(\mathbf{C}^*)\right)/dt = \nabla U(\mathbf{C}^*)\dot{\mathbf{C}}^* > 0 \quad \Leftrightarrow \quad d\left(\mathbf{P}\mathbf{C}^*\right)/dt = \dot{\mathbf{P}}\mathbf{C}^* + \mathbf{P}\dot{\mathbf{C}}^* > 0\,.$$

We have shown in this paper that welfare stock improvements can be indicated by real NNP flow changes locally in time. However, unless $Y(t)$ is monotone between t' and t'', it does not necessarily follow that $Y(t') < Y(t'')$ indicates that welfare stock is higher at time t'' when compared to an earlier point in time, t'. The underlying reason is that the consumption bundle used as weights in a Divisia price index changes continuously over time. Even though an increase in $\mathbf{P}(t)\mathbf{C}^*(t)$ means that *instantaneous* well being – i.e. utility – increases at time t, such a local result does not translate easily or directly into a general statement for making global welfare stock comparisons.

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